ANALYSIS OF EXPERIMENTAL DATA ON FATIGUE CRACK DEVELOPMENT

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One of the major characteristics of a material is its rupture strength, which determines the size of the flaw bringing about failure at a given stress level [1]. However, the growth rate of a fatigue crack may prove to be the limiting factor in strength calculations under variable load conditions, rather than the rupture strength directly. Because of this, the phenomenon of fatigue growth of cracks under cyclic loading has been studied by many authors over the past fifteen years. A comparison of the theory of the growth of fatigue cracks [2] and available experimental data is presented below.

1. Phenomenological Description of the Developmental Process of Fatigue Cracks. An elastoplastic model of the body was invoked to account for the development of cracks under cyclic loading, and the fine structure of the tip of the crack has been examined [2]. The application of general considerations in dimensional analysis and the Irwin–Orovan energy concept extended to the case of nonstationary development of cracks has made it possible to derive the following dependence for the growth rate of a fatigue crack:

$$\frac{dl}{dn} = -\beta\left(\frac{N_{\max}^2 - N_{\min}^2}{K_c^2} + \ln\frac{K_c^2 - N_{\max}^2}{K_c^2 - N_{\min}^2}\right) \qquad (1.1)$$

Here β and K_c are some constants of the material, n is the number of loading cycles (which plays the role of time in these problems), l is the length parameter of the crack, and $N_{\max}$ and $N_{\min}$ are the greatest and least value of the stress intensity factor during a cycle at the time n. In most cases the value of K_c is close to the rupture strength K_{Ic}, with the exception of those cases where the failure takes place in the course of a small number of cycles, or in the case of thin plates (in the latter case K_c is a function of the plate thickness). The constant β has the dimensionality of length and characterizes the increment in crack length in cyclic loading (in order of magnitude this constant is equal to the increment in crack length as N increases from N to K_c). The constants K_c and β must be determined on the basis of experimental data. Curves 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 in Fig. 1 were plotted on the basis of Eq. (1.1) at $N_{\min} = 0$ and correspond to the following values of the parameter β (in mm): 0.01, 0.03, 0.05, 0.1, 0.2, 0.4, 0.8, 2.0, 4.0, 8.0. The abscissa scale is logarithmic, and 1 mm/cycle is taken as the unit measurement. This plot grid is used in what follows in order to effectively determine the constants β and K_c by the method of superposition.

The dependence of the rate of crack growth on the applied loads is of paramount interest for calculating strength under cyclic loading. It has been studied experimentally by many authors; we analyze only those for which the data are processed in invariant variables (i.e., in N and in dl/dn). The data in the remaining papers proved insufficient for comparison with the theoretical dependence (1.1).

2. High-Cycle Fatigue Cracks. One of the first experimental papers on the study of the growth of crack length under cyclic loads, in which the relationship between the rate of crack growth and the stress intensity factor has been investigated, was the work of Donaldson and Anderson [3]. This paper contains a wealth of material obtained on the basis of a study of specimens with central and lateral through cracks (stresses were applied both to the edges of the plate and to the edges of the crack). Experiments carried out on alloys of aluminum, nickel, magnesium, steel, etc. attest to the existence of some correlation between dl/dn and $N_{\max}$. The clearest picture of the phenomenon is given by experiments carried out on the

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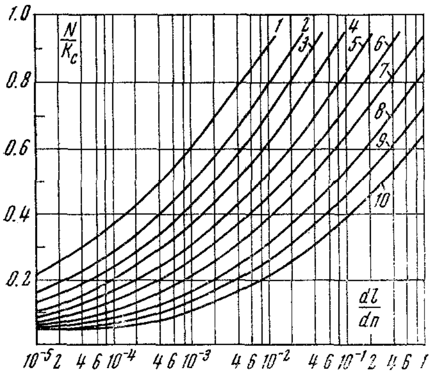

Fig. 1

aluminum alloys 2024-T3 and 2024-T6. The ratio $\sigma_{min}/\sigma_{max}$ varied roughly from 0.2 to 0 in the experiments staged by Donaldson and Anderson (there were also some negative values). The corresponding values of the ratio N_{min}/N_{max} will clearly vary over the same range and can be neglected on the basis of the dependence (1.1), i.e., it is safe to assume $N_{min} = 0$. By the way, this inference was also made by Donaldson and Anderson on the basis of their experiments. The averaged experimental data reported in [3] are plotted in Fig. 2. The broken curve is plotted for the alloy 7075-T6, the dash segments indicate the approximate spread of points corresponding to N = const for each series of tests and corresponding to the fiducial probability 0.98. The dot–dash curve is plotted for the alloy 2024-T3, and the spread of data points is indicated by the dot–dash segments at the same fiducial probability 0.98. It should be noted that the spread increases as the number of cycles is decreased. Figure 2 also exhibits continuous curves plotted on the basis of Eq. (1.1) at the following values of the constants: β = 0.15 mm, K_c = 90 kg/mm$^{3/2}$ for the alloy 7075-T6 and β = 0.10 mm, K_c = 90 kg/mm$^{3/2}$ for the alloy 2024-T3. These constants were arrived at by superposing the experimental curves and the curves plotted in Fig. 1.

A large number of alloys of aluminum, molybdenum, titanium, etc. have been studied at $N_{min} = 0$, in work reported by Paris [4], Pearson [5], and Johnson and Paris [6]. These authors differed from the authors mentioned previously in presenting their results in the form of several empirical formulas. Paris proposes the empirical relation $dl/dn = CN^4$, which provides an excellent approximation of the experimental data points for the aluminum alloys 2024-T3 and 7075-T6 in the growth rate intervals from 10^{-5} to 10^{-2} mm/cycle. Pearson gives an approximation of the form $dl/dn = CN^{3.6}$ for the same materials, with the pertinent range of growth rates (10^{-4} to 10^{-3} mm/cycle) apparently too narrow to support the derivation of any analytical dependences. Paris' empirical formula was obtained directly [2] from Eq. (1.1) at $N_{max}/K_c \leqslant 0.5$, i.e., for those cases where the number of cycles is comparatively high (and the crack growth rate is low). It is precisely such cases that were encountered in the experiments under discussion.

3. Low-Cycle Fatigue Cracks. Paris' formula is no longer valid in the case of low cycle fatigue cracks (i.e., in cases where the stress intensity factor N_{max} is close to the value of K_c, while the crack growth rate is comparatively large, Paris' formula no longer holds). Various investigators using the power-law approximation have noted an increase in the power exponent in that case.

Carman and Katlin [7] conducted experiments using specimens of the martensite-aging steels 250 and 300 (experimental data for 250 steel are plotted as hollow circles in Fig. 3, while data for 300 steel are plotted there as triangles). The curves plotted on the basis of Eq. (1.1) at the assigned values of the constants β = 0.2 mm, K_c = 710 kg/mm$^{3/2}$ for 250 steel and β = 0.09 mm, K_c = 675 kg/mm$^{3/2}$ for 300 steel, plotted as continuous curves in Fig. 3. The agreement with theory is clearly satisfactory within the given spread of experimental data.

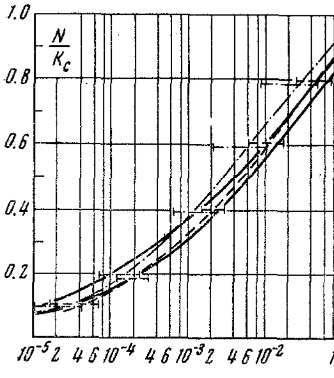

Fig. 2

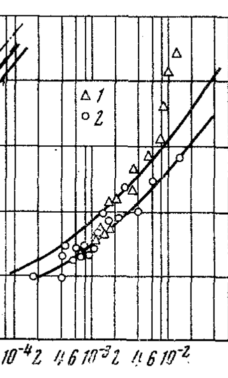

Fig. 3

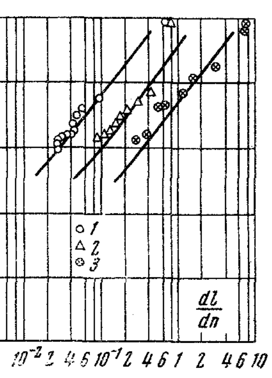

Fig. 4

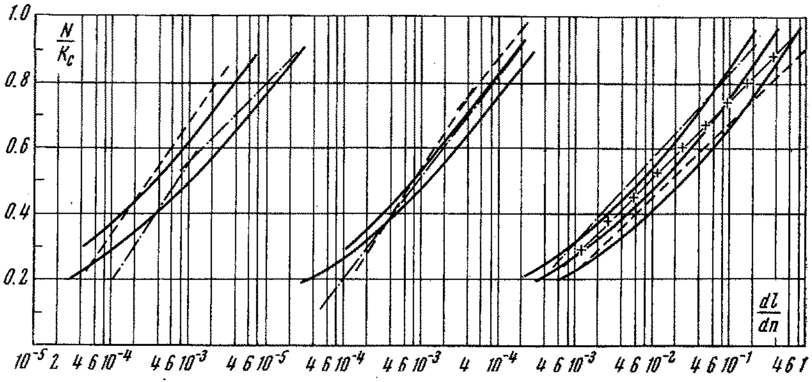

Fig. 5 Fig. 6 Fig. 7

Yang [8] has investigated low-cycle fatigue in several alloys of aluminum and steel. The experimental results are plotted in Fig. 4 (data points are hollow circles in the case of 2024-T6 aluminum alloy, triangles in the case of 310 stainless steel, and crosses in the case of 301 steel). Yang employed the power-law dependence $dl/dn = CN^{\alpha}$ with exponent $\alpha = 5$ for the 2024-T6 aluminum alloy, $\alpha = 7$ for 310 steel, and $\alpha = 7$ for 301 steel, in order to approximate the experimental points. Upon comparing these experimental results and the theoretical curves, we find that Eq. (1.1) provides an excellent description of these data at the following values of the constants: $\beta = 0.34$ mm, $K_c = 192$ kg/mm$^{3/2}$ in the case of aluminum alloy 2024-T6, $\beta = 1.0$ mm, $K_c = 463$ kg/mm$^{3/2}$ in the case of steel grade 310, and $\beta = 4.0$ mm, $K_c = 700$ kg/mm$^{3/2}$ in the case of steel grade 301. The theoretical curves are plotted as continuous curves in Fig. 4.

According to Clark's data [9], the growth rate of a fatigue crack in a specimen of 7079-T6 aluminum alloy (plotted as broken curve in Fig. 5) and Ni–Mo–V steel alloy (dot–dash curve in Fig. 6), over the range $10^{-5} \leq dl/dn \leq 10^{-3}$ is proportional to the third power of the stress intensity factor. Over that range Clark's results are found to closely approximate the curves (1.1) (see continuous curves in Figs. 5 and 6) at the assigned values $\beta = 0.01$ mm and $K_c = 125$ kg/mm$^{3/2}$ for the aluminum alloy 7079-T6, and $\beta = 0.03$ mm, $K_c = 520$ kg/mm$^{3/2}$ for the Ni–Mo–V steel alloy.

Clark used a broken curve (dot–dash curve in Fig. 5) in his investigation of specimens of 5456-H321 alloy to approximate the experimental data points. Clark proposed, as the power exponent in the power-law dependence:

$$\alpha = 2 \text{ for } N / K_c \leqslant 0.6 \quad \alpha = 5.2 \text{ for } N / K_c > 0.6$$

Clark's data are described quite well by the dependence (1.1) at the following values of the constants: $\beta = 0.03$ mm, $K_c = 163$ kg/mm$^{3/2}$ (continuous curve in Fig. 5). For the steel HP 9-4-25, Clark proposed the assigned α values:

$$\alpha = 2.6 \text{ for } N / K_c \leqslant 0.8 \text{ and } \alpha = 9 \text{ for } N / K_c > 0.8$$

(broken curve in Fig. 6). The corresponding theoretically predicted curve plotted on the basis of Eq. (1.1), at $\beta = 0.02$ mm and $K_c = 460$ kg/mm$^{3/2}$, is plotted as a continuous curve in Fig. 6.

V. N. Markochev [10] has proposed an entirely different dependence of the crack growth rate on the stress intensity factor for the range of growth rates $10^{-3} < dl/dn < 1$. On the basis of his own experiments carried out with alloys D167, D16T-1, and V-95, that author proposed a dependence of the form $dl/dn \sim A + \exp(BN_{max})$. The approximation of the experimental data points of this dependence proposed by Markochev is plotted in Fig. 7 (dot-dash curve for alloy D16T, broken curve with crosses for alloy D16T-1, and broken dash curve for alloy V-95). A comparison with theoretical data based on Eq. (1.1) (continuous curves in Fig. 7) reveals fairly close agreement between those data at the assigned values of the constants: $\beta = 0.17$ mm, $K_c = 200$ kg/mm$^{3/2}$ for alloy D16T, $\beta = 0.32$ mm, $K_c = 200$ kg/mm$^{3/2}$ for alloy D16T-1 and $\beta = 0.6$ mm, $K_c = 200$ kg/mm$^{3/2}$ for the alloy V-95.

Analysis of the experimental data on the development of fatigue cracks confirms, within the limits of experimental error, the theoretical dependence (1.1) over a wide range of numbers of cycles to failure.

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